Quantitative investigation of Fresnel reflection in the electromagnetism laboratory

M. Dekker, A. J. Kromminga, and D. A. Van Baak
Department of Physics, Calvin College, Grand Rapids, Michigan 49546

O. J. Pilskalns and J. P. Jacobs
Department of Physics and Astronomy, University of Montana, Missoula, Montana 59812

(Received 15 October 1998; accepted 25 November 1998)

We have developed an experimental apparatus that makes it possible for students to make detailed quantitative measurements of the Fresnel reflection coefficients for the reflection of light from a plane interface. The experiment is made possible by the linear polarization, and impressive power stability, exhibited by visible and near-infrared diode-laser modules, and it has been a very effective addition to our sophomore electromagnetism laboratories.

I. INTRODUCTION

When a beam of light encounters a plane interface between two dielectric media, it generates outgoing reflected and refracted beams. For at least three centuries, students have been made familiar with the *directions* of these outgoing beams, and have had opportunity to investigate the familiar laws of reflection and refraction that relate the angles involved. But few students have been able to investigate quantitatively the *intensities* of the outgoing beams, despite the rather dramatic angular and polarization dependencies of these intensities. We report here an experiment that makes accessible to sophomore students in electromagnetism courses not only the familiar ''kinematic'' properties of the angles of reflection and refraction, but also ''dynamic'' properties such as the reflection coefficients. The experiment reveals to students the importance of unexpected polarization phenomena in a familiar environment, and the data it produces can be compared in detail with quantitative predictions of historically important theories of light. The experiment also introduces students to some up-to-date techniques in optics, and adds some refreshing novelty to an otherwise too-familiar investigation of well-established laws.

The experiment we report here has been made possible by the availability of low-cost visible and near-infrared diode-laser sources which deliver beams of light with good collimation, very impressive power stability, and intrinsic linear polarization. We have adapted disused force tables, spectrometer tables, and optical elements to provide an arena for the reflection and refraction phenomena, and have used low-cost silicon photodetectors to permit quantitative measurement of the relative powers of the incoming and outgoing beams.

The theoretical computation of reflection coefficients for light at a plane interface is a standard exercise in upper-level electromagnetism texts,[1] and methods for generalizing the computation have appeared in this journal.[2] Though the measurement of Fresnel reflection coefficients has become the basis of the whole field of ellipsometry, there are few experiments adapted for student use that introduce this part of electromagnetic theory; one exception in this journal is Driver's paper[3] describing a method of using a wide-area beam of sodium D-light to deduce the angular dependence of the Fresnel reflection coefficients. This paper is a modern laser realization of the same goal, and we will show that the use of diode-laser techniques materially simplifies the data-taking, and the conceptual accessibility, of the experiment we describe.

In this paper we review in Sec. II the history, and the results, of electromagnetic theory's prediction for partial reflection of light at a plane dielectric interface. We describe in Sec. III the apparatus and methods we have used to make these theoretical predictions accessible to experimental investigation by undergraduates, and in Sec. IV we present some of the possible results that can be readily obtained. In Sec. V we contemplate other capabilities of the apparatus, and summarize our results.

II. THEORY

Though nearly every student knows that light can be *refracted* when crossing from one transparent medium into another, some students think that the entire transparency of both media forbids any *reflection* at such an interface. Nevertheless the myth of Narcissus makes it clear that the phenomenon of partial reflection of light at the plane interface between dielectric media has been known for millennia. The degree to which light is reflected at such an interface was the subject of a great deal of discovery and research in 19th-century physics, and it might be profitable here to review the chronology of discovery that lies behind what are now called the Fresnel reflection coefficients.

We may begin with the invention, by Thomas Young in 1802, of a wave theory of light to account for ''Newton's rings,'' and the discovery, by Malus in 1809, of the phenomenon he called ''polarization'' of light. It is interesting to note that the production of (partial) polarization of light in Malus's observations was due to oblique reflection at an air–glass interface, and interesting too to note that these phenomena were immediately interpreted in terms of a transverse wave theory. By 1815 Brewster had announced the total polarization of light reflected at the angle since named after him,[4] and had also formulated some empirical formulae for the reflection coefficients at general angles. The Fresnel formulae still known by that name were published in 1825,[5] and were derived on the assumption that light could be modeled as the propagation of a single transverse displacement in an elastic solid, the ''luminiferous ether.''[6]

In fact Fresnel's derivation was not the result of a fully mechanical theory applied to the elastic-solid hypothesis; rather, in the words of E. T. Whittaker,[6]

> His method was to work backwards from the known properties of light, in the hopes of arriving at a mechanism to which they could be attributed... . The 'displacement' of Fresnel could not be a displacement in an elastic solid of the usual type, since its normal component is not continuous across the interface between the two media.

Instead, the two boundary conditions that Fresnel used are equivalent to the assumption (1) that the tangential component of the displacement is continuous across the interface, and (2) that energy is conserved at the interface.

Nevertheless the theory gave the now-familiar results: Given a beam of light propagating from one medium of refractive index n_1 to another of refractive index n_2, and forming angles (measured with respect to the normal) of θ_1 and θ_2 in the two media, then the fraction of energy reflected at the interface is predicted to be the reflection coefficient $R(\theta_1,\theta_2)$, whose form depends on the orientation of the ''displacement'' with respect to the plane containing the incident, reflected, and refracted beams. In the case that the displacement is assumed to be perpendicular to the plane containing all the beams (s polarization), the prediction is

$$R_s(\theta_1,\theta_2)=\left(\frac{\sin(\theta_1-\theta_2)}{\sin(\theta_1+\theta_2)}\right)^2=\left(\frac{n_1\cos\theta_1-n_2\cos\theta_2}{n_1\cos\theta_1+n_2\cos\theta_2}\right)^2, \tag{1}$$

whereas if the displacement is assumed to lie in the plane containing all the beams (p polarization), the prediction is

$$R_p(\theta_1,\theta_2)=\left(\frac{\tan(\theta_1-\theta_2)}{\tan(\theta_1+\theta_2)}\right)^2=\left(\frac{n_2\cos\theta_1-n_1\cos\theta_2}{n_2\cos\theta_1+n_1\cos\theta_2}\right)^2. \tag{2}$$

Here the angles θ_1 and θ_2 are not independent, but are related by Snel's law,

$$n_1\sin\theta_1=n_2\sin\theta_2. \tag{3}$$

The form of Eq. (2) immediately explains Brewster's law, since at the angle defined by

$$\theta_1+\theta_2=\frac{\pi}{2}, \tag{4}$$

the reflection coefficient for the p polarization is predicted to vanish. It is not clear when the quantitative predictions of the reflection coefficients, Eqs. (1) and (2), away from Brewster's angle were first confirmed to contemporaries' satisfaction.[7,8,9] What is clear is that the matter was regarded as settled long before the formulation of Maxwell's electromagnetic theory for light. This was published in finished form only in 1864, and led to the celebrated prediction for the velocity of light in free space; but it did not lead immediately to a rederivation of the Fresnel equations.[10] As Whittaker says, (see Ref. 6, Chap. VIII):

> It may seem strange that Maxwell, having successfully employed his electromagnetic theory to explain the propagation of light in isotropic media, in crystals, and in metals, should have omitted to apply it to the problem of reflection and refraction.

Once again, the problem lay in boundary conditions; Maxwell ''was not able to satisfy himself regarding the conditions which should be satisfied at the interface between the media.'' Textbooks nowadays derive these boundary conditions directly from Maxwell's equations (continuity of the tangential components of the fields E and H, and of the normal components of D and B), where the electric field of the light plays the role of Fresnel's ''displacement;'' then assuming that E of the light lies perpendicular to the plane containing the light beams yields Eq. (1), while assuming E lies in that plane yields Eq. (2).

It is worth pointing out to students that the Fresnel formulae which are the subject of this laboratory exercise were first derived from Fresnel's transverse-displacement theory which we now regard as supplanted, and that the successor transverse-electromagnetic theory of Maxwell made no new prediction for the coefficients. To knowledgeable students this will help to illustrate the problem of the ''underdetermination of theory by data'' that is of such concern to philosophers of science.

III. APPARATUS

The goal of this experiment is to provide to students, typically in sophomore-level laboratory courses in electromagnetism, the apparatus required to measure experimentally the values of the Fresnel reflection coefficients, for one wavelength, over a wide range of angles and for both states of polarization. There are further desiderata, including that the equipment should be reliable, simple, and robust; happily, these goals can all be met at reasonable cost.

A. Light sources

Our experiments depend on beams of collimated light from diode laser sources, of nominal wavelengths near 670 or 780 nm in the red and near-infrared (IR) spectral regions, respectively. The beams carry total power of 3–5 mW, are collimated to a few milliradians' divergence, and have a cross-sectional area of about $2\,\mathrm{mm}\times 4\,\mathrm{mm}$. From a student's perspective, they form a fair approximation to an idealized ''ray'' of light; but they are of sufficient area and collimation that their partial reflection are well approximated by the Fresnel coefficients derived for plane waves of infinite transverse extent. The choice of wavelength is somewhat arbitrary; since the 670-nm laser beams are visible, they make the apparatus easier to align, and the measurements easier to perform. The 780-nm laser beams, because they are nearly invisible,[11] add some complication to the setup and measurement, but also add a certain novelty to the experiments; students tend to enjoy working with the IR beams.

A bit of detail about the properties of diode lasers will help to explain the features of the light beams important to this experiment. Laser radiation in these devices originates in a waveguide structure of transverse dimensions of order $1\,\mu\mathrm{m}\times 3\,\mu\mathrm{m}$, and length about 0.2 mm, buried inside a semiconductor chip in turn mounted inside a windowed package. These laser packages also contain an internal photodiode used to monitor the power output of the laser. The electromagnetic wave that propagates inside the waveguide structure is linearly polarized, so the laser output is nearly 100% linearly polarized at birth. But because of the tiny transverse dimensions of the waveguide structure, the output diverges dramatically and unsymmetrically, with angular full-width at half-maxima of order $20°\times 60°$ in two transverse angles.

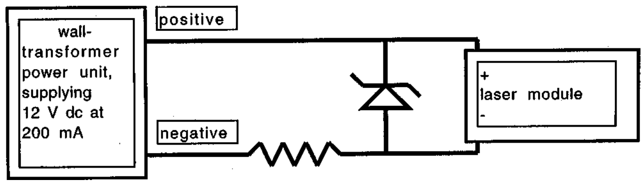

Fig. 1. The electrical circuit used to operate a visible diode-laser module from a wall-transformer power source. The wall transformer delivers an unregulated dc supply at the required current of about 60 mA; the resistor and the 5.1 V Zener diode regulate this to some extent. The resistor is placed in the negative lead because the positive lead of visible diode-laser modules is typically in common with the module's case.

Thus diode laser sources are nearly always placed at the focal point of a lens of short focal length, typically yielding a beam well collimated in angle, but of elliptical cross section. The character of the waveguide mode inside the diode laser is such that the electric field vector of the light lies along the short axis of that ellipse; this is not in any sense obvious to students, and in fact the experimental data on Fresnel reflection, together with an interpretation based on Maxwell's theory, provides the empirical confirmation of this statement.

The output wavelength and power of such a diode-laser source depend on both the temperature of, and the dc current passing through, the semiconductor chip. There are two ways often used to control a diode laser. First, if the diode laser's temperature and current are both kept controlled, then both the output power and wavelength will be stable; this is the common control system used when both the intensity *and* wavelength of the laser must be controlled precisely. A simpler and more direct alternative chosen for our experiments uses the signal from the internal photodiode to stabilize only the output power of the laser, using a feedback loop to vary the diode-laser current to compensate for changes that would otherwise be caused by temperature (this is known as automatic power control or APC).[12] This choice leaves the diode-laser wavelength subject to temperature changes, but the rate of shift is only about 0.3 nm/°C, which is perfectly tolerable for this experiment. In either method, the diode-laser output may also possess multiple longitudinal modes, spaced about 0.2 nm away from the dominant wavelength, but again for the purposes of this experiment, the output may be taken to be monochromatic. The actual wavelength emitted may also be several nanometers longer than the nominal wavelength of the device; students may in fact be motivated to measure the actual wavelength by a suitable diffraction-grating experiment.

Since diode-laser modules are now familiar items of retail commerce, it is worth mentioning a few features of the systems used here. The modules commercially available have been designed to accept power from a dc source of 3–6 V, and are often engineered to be supplied by batteries (as in common laser pointers); but since batteries' finite lifetimes are a nuisance, we have instead arranged to power the modules by wall-transformer units as shown in Fig. 1. Diode laser devices are notoriously vulnerable to damage by reverse voltage, or excessive current, and even when protected by the APC circuits hidden inside the modules they can still be damaged by transients; this is the motivation for the simple Zener-diode circuit shown in Fig. 1. Since the modules are typically cylindrical in cross section, we have built aluminum blocks of square cross section, and mounted the laser modules inside with their polarization vectors lying parallel to one pair of edges of the square.[13] Thus packaged, the modules are robust enough for students' use; the focusing lens in front is left adjustable, but the power setting is fixed.

It is worth mentioning here why familiar helium–neon lasers are not good choices for optical sources in this experiment. These devices operate on an atomic neon transition with wavelength fixed at 633 nm, and emit a beam of cylindrical symmetry and good optical quality, but they do *not* display constant output power. Typical HeNe lasers are ''unpolarized,'' and in fact display a time-varying mixture of two linear polarizations; HeNe lasers forced to operate in one linear polarization (by a Brewster plate internal to the laser cavity) are also available, but both types display competition among multiple longitudinal modes. As a result, the output power displays a marked time variation, typically of order $\pm 10\%$ on a time scale of about a minute, related to the thermal expansion of the cavity. Naturally, one could use a two-detector geometry in an experiment, and normalize out these variations, but that would considerably complicate the experiments discussed here. In addition to these essential difficulties, HeNe lasers are also disadvantageous in their greater bulk, expense, and fragility, leaving diode lasers as the clear choice for light sources in these experiments.

B. Light detection

The experiments discussed here require that the power in various optical beams be measured. Happily, measurement of the absolute power is not required, since the theoretical predictions for reflection coefficients are the ratios of two measured powers; thus expensive laser power meters are not required (although a calibrated power meter is handy for initially setting the power output of the laser). All that is needed is a device which is polarization and wavelength insensitive, but which produces an output proportional to incident optical power. This need is very well met by a simple silicon *p-n* junction photodiode (better known in its application as an energy-generating ''solar cell''). Under illumination, such devices produce a short-circuit current which is very accurately linear in the optical power absorbed, with a conversion efficiency described very nearly by one electron–hole pair produced per photon absorbed. The main inefficiency is the loss, to reflection, of the incident light; but if the photodiode is used near normal incidence, this loss is a polarization-independent constant fraction of the incident light. As a result, a given photodiode can be treated as a converter from optical power to dc electrical current, with a fixed and understandable conversion ratio.

In our experiments we have used simple and inexpensive unencapsulated photodiodes with large enough active area to capture on the photodiode the whole of the laser beam, so that we measure total power rather than power per unit area.[14] We incorporate these photodiodes directly into a metal structure that clamps them into proper position on the optical tables we've chosen. Our devices display a conversion efficiency of about 400 μA (550 μA) of current per milliwatt of incident optical power, representing about 74% (87%) quantum efficiency for the 670-nm (780-nm) system. Using diode laser modules of under 5-mW total power yields dc currents in the 0–2 mA range. Thus the entire ''detector circuit'' consists of attaching the photodiodes to the input of a digital multimeter used as a 2-mA dc ammeter; there is no amplification, bias supply, zero offset, or power source needed. Our digital multi-meters have (on this range) an input impedance of 100 Ω, small enough that the meter displays very nearly the current that the photodiode would deliver in a true short circuit.[15]

With this detector setup, it is easy to verify that the power stabilization of commercial diode lasers is very good indeed; for the modules we've used, with the power supplies noted above, the signal resulting from sending the full laser beam into the detector is typically near 1.4 mA; after a few minutes' warm-up, this current exhibits a stability of $\pm 1\ \mu$A over a time scale of seconds to hours.[16] Thus there is no need, in our experiments, for frequent checks of the ''full scale'' reading of the lasers. As a further convenience, we find that, under perfectly adequate illumination of our laboratory by fluorescent lighting, the ''background'' light intensity on the detectors produces a signal of only 10–20 μA. This represents less than 1% of the total light signal, but since the precision obtainable is higher still, and since this zero offset may vary as the detectors are aimed in different directions, we instruct students to take readings in pairs, differing only in that the laser beam is blocked near the laser source for one of the readings.[17]

C. Angle measurement

A final part of our experimental apparatus is the arena in which we hold the laser source, the optical element, and the detector. In the 670-nm experiment, we have adapted some old force tables of diameter 40 cm for this purpose, since these are marked with a 0–360° angular scale easily read to an accuracy of about 0.5°. To hold the laser sources, we have built ''arm'' structures which rotate about the vertical shaft supporting the horizontal tables; these allow the angle of incidence of the optical beam to be varied at will, meanwhile ensuring that the laser beam remains aimed at the center of the table, and propagates horizontally about 5 mm above the table surface. On the table surface, we can place a variety of optical elements, insuring that the plane interface illuminated by the incoming beam lies right at the center of the optical table. At the edge of the table, we can position white paper cards (to note the angular position of incoming or outgoing laser beams), or clamp the detector structures mentioned above (to measure the optical power).

In the 780-nm experiment, we use an old spectrometer table with peripherals removed which has been adapted for this experiment (see Fig. 2). It was chosen mainly because it provides a convenient and precise means of measuring the angles. The slit and objective arm have been replaced by a 60-cm-long aluminum arm which holds the laser light source described above. The eyepiece arm has been replaced by a second aluminum arm, also 60 cm long, which holds the light detector. As the detector arm is rotated about the center of the spectrometer table, the angles can be read with 0.03 deg precision from the vernier scale which is inscribed in the base of the spectrometer. The spectrometer's prism has been replaced by the dielectric plane surface, which is a 1-in.-diam by 1-cm-thick disk of quartz, Pyrex™, Plexiglas™, or other material held in place on the prism platform by a two axis mirror mount which facilitates setup and adjustments. The relatively large thickness was chosen to allow for a clear differentiation between reflections from the front and back surfaces of the disk.

For both experiments, we view it as an important pedagogical point that the entire experimental arena is free of any

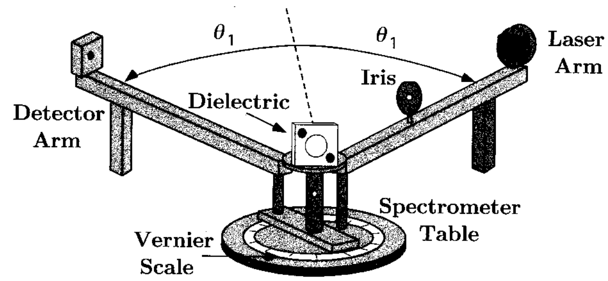

Fig. 2. Experimental apparatus for the Fresnel reflection experiment using the 780-nm diode laser. An old student laboratory spectrometer has been adapted for this purpose. It provides a convenient and precise means of measuring angle of incidence for the laser beam. The dielectrics used are quartz, Pyrex, or Plexiglas. For each measurement, the position of the laser is kept fixed, the detector is placed at the desired angle of incidence, and the dielectric is rotated to give maximum intensity.

device that looks like the ''Polaroids'' that students associate too exclusively with polarization phenomena. Naturally the laser sources are producing nearly 100% polarized light, but this is not at all obvious to the naive user; naturally the oblique incidence of this light on a plane interface produces angle- and polarization-dependent reflection coefficients, but we leave the discovery of these facts as a surprise for the students.

D. Safety

Finally, it is worth considering the ocular-safety issues associated with this laboratory. Compared to the familiar 633 nm wavelength of HeNe lasers, the human eye is less sensitive[18] to radiation at 670 nm by a factor of about 5, and at 780 nm by a factor of 12 000, so that the sources used in these experiments are more powerful than they appear to be. Nevertheless it is easy to ensure that the laser sources can be used with safety, provided only that all users are instructed (and all apparatus is arranged) to avoid pointing a collimated beam into anyone's eye. The applicable safety standard[19] at 670 nm for exposure of indefinite duration is an upper limit on flux of 63 μW/cm^2: Thus a full laser beam of over 3 mW falling on an eye is clearly unsafe; but if that same 3-mW beam falls onto a white card which diffuses the light over 2π steradians, the bright spot is safe for viewing of indefinite duration even at a card-to-eyeball distance of 3 cm. Beams at 780 nm fall into a different safety classification, and here the applicable safety standard for exposure of indefinite duration is an upper limit on flux of 465 μW/cm^2: Thus a full laser beam of over 3 mW falling on an eye is again unsafe; but if that same beam falls onto a white card which diffuses the light over 2π steradians, the bright spot is safe for viewing of indefinite duration even at a card-to-eyeball distance of only 1 cm. To meet these safety standards, we arrange the optical tables such that all the beams involved stay in a horizontal plane well below chest height, we warn students about the hazards of specular reflecting surfaces, and we instruct students on the safety issues involved, so as to prevent the direct encounter of the collimated laser beams with anyone's eyes.

IV. RESULTS

We now describe two kinds of experiments that this sort of experimental apparatus makes possible: The first requires a special sort of optical sample, and takes angular data for refraction, and intensity data for reflection; the second requires only a plane interface, and takes only intensity data in reflection.

The first experiment is motivated by the existence of items common to ''blackboard optics'' kits; the samples used are right semicircular cylindrical prisms made of glass or plastic, and in this experiment they are used with the center of the plane face of the prism placed right at the center of the angular arena. With this geometry (easily aligned by requiring laser light incident from the 0° position to be reflected straight back to the laser), the light refracted into the prism emerges purely radially at the semicircular surface of the prism, and thereby suffers no additional refraction upon exiting. The angular position of the refracted ray can thus be read at the periphery of the angular table. Thus far such an experiment is a traditional test of Snel's law, modernized only by the use of a beam of laser light; the added feature in this experiment is to measure the intensities of the incident and reflected rays.

Our method of choice is to measure the intensity of the incident ray by removing the prism entirely, and allowing the laser beam to propagate across the diameter of the force table to the photodiode detector. The current resulting (corrected for the current generated by room light) is a measure of the optical power that would result from a 100% reflection of the light at a surface at the center of the force table. Now with the prism replaced at the center of the table, we choose not to measure the intensity of the refracted ray (since this is subject to additional Fresnel-reflection losses upon exiting the sample, and absorption losses inside the sample); rather, we measure the intensity of the reflected ray, at the edge of the force table, using the same detector on a beam that has traveled the same distance as in the measurement done on the incident beam. The ratio of photodiode currents (each of course separately corrected for ambient room light) is then a direct measurement of the reflection coefficient of the plane interface, for the wavelength, angle, and polarization used.

The refraction data obtained by this method is wholly traditional, and ranges over angles of incidence from $-70°$ to 70°; we have students plot the data they obtain on various scales, including $\theta_{\text{refracted}}$ vs θ_{incident}, $\tan(\theta_{\text{refracted}})$ vs $\tan(\theta_{\text{incident}})$, and $\sin(\theta_{\text{refracted}})$ vs $\sin(\theta_{\text{incident}})$. Typical data plotted in the last of these ways is shown in Fig. 3. The plot also shows a linear fit, whose slope, according to Snel's law, gives the inverse of the refractive index of the prism for the wavelength used (nominally 670 nm). The resulting index of refraction, $n=1.51\pm0.01$, can then serve as the measured value of the input parameter in the Fresnel equations, and according to Fresnel's theory it ought to predict fully the results for reflection coefficients.

The novel feature of this experiment is the reflection-coefficient data that it provides, and some data for angles of incidence from 10° to 85° are shown in Fig. 4. The two sets of data plotted differ only in the rotation through 90° of the block containing the laser module between the two sets of readings; we find that the students are best surprised if they are guided to take first the data *not* exhibiting the Brewster-angle zero. The quite different results obtained for the two orientations of the laser source illustrate as clearly as possible that effects attributable to polarization are dramatically visible in an experiment that uses no Polaroids or other overt ''polarization components.'' Overlaying the data in Fig. 4 are the curves predicted by Fresnel's theory, evaluated *not* by fitting using any adjustable parameters, but rather by evaluating the Fresnel predictions using the index of refraction deduced from the angular data in the refraction measurements. The agreement between the theory and the measurements is intended to show students that a single parameter, called the index of refraction, fully characterizes not only angular properties of refraction, but also intensity properties of reflection. The apparatus described is clearly able to measure reflection coefficients over the wide range of 0.005 to 0.5 or higher, and the measured coefficients for one of the polarizations clearly support the existence of a null reflection at Brewster's angle. The detailed agreement between data and theory is not perfect, however; the reflection coefficients measured are systematically a bit smaller than those predicted by theory. Nevertheless the agreement is quite striking, given that Fig. 4 is a comparison between data and theory using no free parameters. We attribute the imperfections of the agreement between data and theory to the imperfection of the glass prism used; the plane surface of the prism

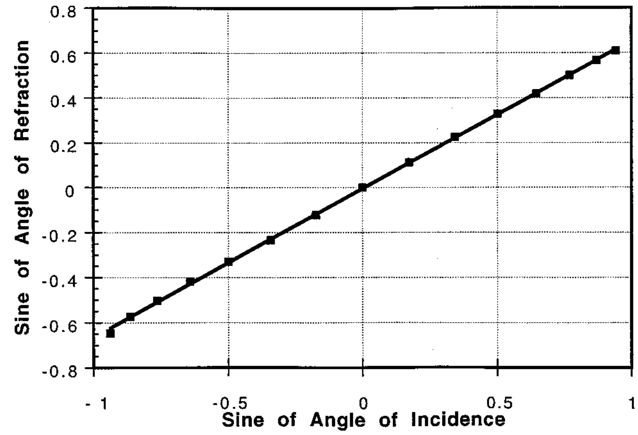

Fig. 3. Angular data obtained using 670-nm laser-beam illumination of a right semicircular prism, plotted so as to reveal Snel's-law behavior. The slope of the best-fit line (0.660) gives the ratio of indices of refraction of air and of the glass of which the prism is made.

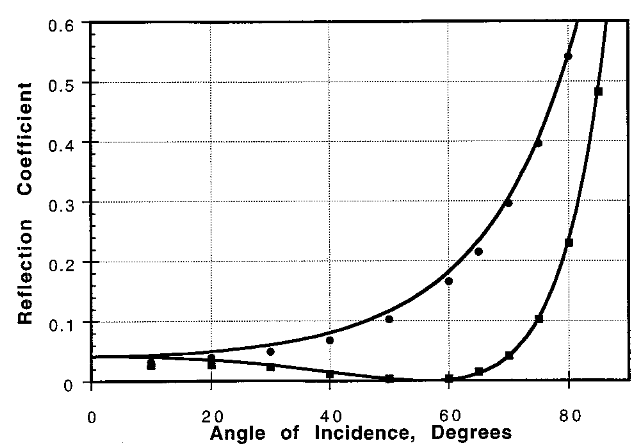

Fig. 4. Reflection-coefficient data obtained using 670-nm laser-beam illumination of the plane interface between air and the glass prism used in Fig. 3, plotted as a function of the angle of incidence, for two orientations of the (linearly polarized) laser source. Overlaying the data are curves generated from the Fresnel reflection formulae, evaluated not by best fit but rather by using the index of refraction deduced from the refraction data shown in Fig. 3.

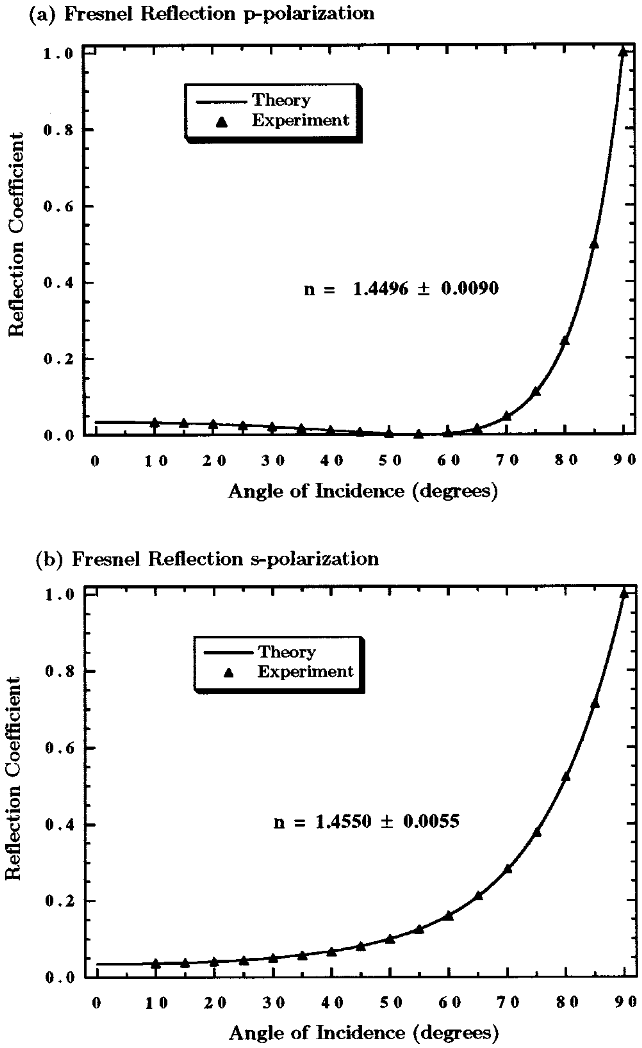

Fig. 5. Typical experimental results for the Fresnel reflection experiment with $\lambda = 780$ nm. The value of n shown on the plots is the result of the one-parameter fit described in the text. The theory curve (the solid line) is generated using the fit value for n. The triangles show experimental values. In both cases the actual index of refraction ratio is $n/n_{air} = 1.4540$ (where n_{air} was measured in a separate experiment). (a) Results for p polarization (electric field parallel to the plane of incidence). (b) Results for the s-polarization mode (electric field perpendicular to the plane of incidence).

is not polished to optical-imaging standards, and striations in the surface cause visible imperfections in the reflected beam, so that it is plausible to attribute to scattering some of the deficiencies in the measured reflection coefficients.

The second sort of experiment involves no refraction data at all, and obtains only reflected-intensity data, making use of any sample with one plane surface. Again the apparatus makes it possible to measure, at the wavelength of the laser used, and for the two orientations of the laser source, the reflection coefficients applicable at the air–sample plane interface. All that is required of the sample is that any reflections from its back face, or other faces, be separable from the main reflection from its front face, and that its face be large enough fully to contain the laser beam's area even when illuminated obliquely. A wide range of samples fulfill these conditions, including exotic ones such as semiconductor wafers, gem stones, nonisotropic materials such as calcite, etc. In Fig. 5, we show a typical data set for this experiment from the 780-nm setup obtained for reflection from the front surface of a 1-cm-thick disk of quartz. The solid curves on these plots are the results of one-parameter fits to the experimental data using the software program MAPLE (version 3). Data have been normalized by the intensity measured at $\theta_1 = 90°$ with the quartz disk removed. The major sources of error in the measurement of the relative intensity come about from uncertainties in the angle of incidence, and fluctuations in the laser intensity, which when combined result in an uncertainty of about 0.5% in the relative intensity. We have found that a straightforward method for performing this nonlinear fit which is accessible to undergraduate sophomores is to introduce the chi-squared function for the curve in question. Rewriting Eq. (1) or (2) in terms of only θ_1 and $n = n_2/n_1$ to obtain $R(\theta_1, n)$, we have

$$\chi^2 = \sum_{i=1}^{N} \frac{(R_i(\theta_1) - R(\theta_1, n))^2}{\sigma_i^2}, \tag{5}$$

where $R_i(\theta_1)$ are the measured values for the relative intensities, σ_i are the uncertainties in those measurements, and N is the number of data points. We let MAPLE take the derivative of this function with respect to n, then using MAPLE to evaluate this derivative, find the value of n for which the derivative is nearly zero, minimizing the deviations between the fit and the experimental data. Reduced chi squared, $\chi_\nu^2 = \chi^2/(N-1)$, for these fits are typically between 1.0 and 1.5. The uncertainty in n is given by[20]

$$\delta n = \sqrt{\chi_\nu^2} \left[\sum_{i=1}^{N} \left(\frac{1}{\sigma_i} \frac{\partial R}{\partial n} \right)^2 \right]^{-1/2}, \tag{6}$$

where $R = R(\theta_1, n)$, and the factor in front of the square brackets represents the correction to δn in cases where $\chi_\nu^2 > 1$. In the data shown, the values for index of refraction found are 1.4496 ± 0.0090 and 1.4550 ± 0.0055 (for the p- and s-polarized case, respectively). In both cases the actual index of refraction ratio is $n/n_{air} = 1.4540$ (where n_{air} is measured in a separate experiment). These results clearly illustrate both the precision and the accuracy that can be obtained with this experiment.

There are no doubt many other experiments beyond these two sorts that can be performed with this apparatus; we have, for example, used the semicircular glass prisms to perform experiments in *internal* reflection, in which we can take data both short of, and beyond, the critical angle. We have contemplated reflection experiments on anisotropic materials, on metallic reflecting surfaces, or on sample surfaces with thin dielectric coatings. All of these experiments have a place somewhere in the electromagnetism curriculum, but we have found the experiments of either type described above are accessible to sophomore-level students, and can be readily performed in a 2- or 3-h laboratory period. Since the apparatus required is not expensive, and since the experiments illustrate some features of considerable conceptual and practical importance, we feel that this exercise in Fresnel reflection serves as a valuable addition to our electromagnetism laboratories.

V. CONCLUSIONS

In summary, we have described how the introduction of diode-laser light sources, and simple but very effective photodetectors, into otherwise familiar experiments on reflection and refraction, can revitalize these experiments and materially enlarge the scope of the phenomena they can reveal. We find that an otherwise mundane laboratory exercise measuring the ''kinematic'' angular properties described by the laws of reflection and refraction is changed dramatically when ''dynamic'' intensity properties can also be measured. We find that the use of diode-laser sources offer a good way to surprise students with the unexpected properties that arise in an environment seemingly lacking polarization components. We find that photodiode detectors offer an easy way to make measurements of light intensity, and that a simple apparatus is sufficient to provide an arena in which the details of the Fresnel equations for reflection coefficients can be studied. At rather low cost, an otherwise aged experiment too easily relegated to ''ray optics'' can be turned into a slightly glamorous laser-based demonstration of unexpected polarization properties of light and a test of some of the dynamical predictions of Maxwell's theory; given the importance of electromagnetic theory in physics, engineering, and technology, this seems to us to be an important pedagogical improvement.

ACKNOWLEDGMENT

These experiments are outgrowths of support from the National Science Foundation for two workshops in diode-laser experiments under Grant Nos. DUE-9255528 and DUE-9455064.

[1]For example, see P. Lorrain and D. R. Corson, *Electromagnetic Fields and Waves* (Freeman, San Francisco, 1970), 2nd ed., pp. 508–519 or D. J. Griffiths, *Introduction to Electrodynamics* (Prentice–Hall, Englewood Cliffs, NJ, 1989), 2nd ed., pp. 363–368.

[2]Edward Collett, ''Mueller–Stokes matrix formulation of Fresnel's equations,'' Am. J. Phys. **39**, 517–528 (1971); William T. Doyle, ''Graphical approach to Fresnel's equations for reflection and refraction of light,'' *ibid.* **48**, 643–647 (1980); R. K. P. Zia, ''Symmetric Fresnel equations: An energy conservation approach,'' *ibid.* **56**, 555–558 (1988).

[3]H. S. T. Driver, ''An undergraduate experiment to measure the reflectances of a dielectric surface,'' Am. J. Phys. **46**, 696–699 (1978).

[4]D. Brewster, ''On the laws which regulate the polarization of light by reflexion from transparent bodies,'' Philos. Trans. R. Soc. London **105**, 125–159 (1815).

[5]A. J. Fresnel, ''[Extrait d'un Mémoire] sur la loi des modifications imprimées à la lumière polarisé par sa réflexion totale dans l'intérieur des corps transparents,'' Ann. Chim. (Paris) **29**, 175–87 (1825). Also in *Oeuvres Complètes D'Augustin Fresnel* (Imprimérie Impériale, Paris, 1866–1870), Vol. 1, pp. 753–762.

[6]A survey of this history appears in E. T. Whittaker, *A History of the Theories of Æther & Electricity* (Dover, New York, 1989), Chap. IV.

[7]The equations were verified at normal incidence by Arago in the visible and by Provostaye and Desains in the IR (Ref. 8, p. 363). In Ref. 9 on p. 392, Buchwald states: ''[The Fresnel equations]... could not in any case be tested themselves, since accurate photometric techniques were not available... .'' Certain implications of the equations were tested, however, and by 1830 they were generally accepted as empirical rules without a proper theoretical basis (Ref. 9).

[8]Thomas Preston, *The Theory of Light* (Macmillan, London, 1912), 4th ed.

[9]Jed Z. Buchwald, *The Rise of the Wave Theory of Light* (The University of Chicago Press, Chicago, 1989).

[10]The derivation was first performed explicitly by H. A. Lorentz in 1875 (see Ref. 9, p. 392).

[11]IR viewing cards can be purchased for only a few dollars at many electronics vendors including Radio Shack.

[12]For the 670-nm experiment we used commercial diode-laser modules, consisting of diode-laser package, collimating lens, and power-conditioning board (Beta Electronics M2670, though DigiKey 11043-ND might be more accessible). For the 780-nm experiment we used a Sharp 5-mW (LT022MD) laser driven with an APC circuit module made by Thor Labs (LD1100). The laser is mounted in a 9-mm type collimating tube with collimating lens (similar to Thor Labs LT110P). The main desiderata of both systems are APC and an adjustable focusing lens.

[13]In the 670-nm system we achieve this alignment by rotating the module within the block at such an angle as to minimize the optical power reflected at an interface encountered near Brewster's angle. Alternatively, as in the 780-nm system, one can mount the collimating tube in a rotation stage (Thor Labs RSP1) which allows for precise rotation of the entire laser head to adjust the plane of polarization of the incident light. The rotation stage is in turn mounted on a two-axis mirror mount (Thor Lab KM1) to allow for precise steering of the laser beam. More advanced experiments can be performed with the laser set to arbitrary polarization angle, in which case the reflected intensity can be predicted by proper combination of Eqs. (1) and (2).

[14]United Detector Technology S100-VL with 1-cm^2 active area, or Thor Labs FDS100 Silicon PIN diodes with 13.7-mm^2 active area. For the smaller active area diodes we use an iris diaphragm (Thor Labs ID12) after the laser to reduce the beam size so that it fits on the detector.

[15]A current of 1 mA from a photodiode will create a 100-mV potential difference across such a meter, and the photodiode; but because of the shape of the current–voltage curve for an illuminated photodiode, this creates an error of order 1 μA or less in the measured current. Alternatively, one can connect the photodiode to an op-amp current-to-voltage converter and measure the output with a voltmeter. This removes any current errors of the type described above, since the photodiode anode is at virtual ground. This method is used in the 780-nm experiment.

[16]The largest systematic error is not drift in the optical power received, but point-to-point variations in the photodiode sensitivity. Purpose-crafted photodiodes (unlike solar cells) keep such sensitivity variations under 1%, but it is still worthwhile to keep the laser beam aimed at the center of the photodiode.

[17]This is preferred to turning the laser on and off electrically, since blocking the beam does not disturb the temperature equilibration of the laser power.

[18]David H. Sliney *et al.*, ''Visual sensitivity of the eye to infrared laser radiation,'' J. Opt. Soc. Am. **66**, 339–341 (1976).

[19]*Laser Safety Guide* (Laser Institute of America, 1991) presentation of the Z136.1 guidelines for maximum permissible exposure for direct ocular exposure.

[20]P. R. Bevington, *Data Reduction and Error Analysis in the Physical Sciences* (McGraw–Hill, New York, 1969).

THE MARK OF AN EXCELLENT TEACHER

Finally, I am not saying that every student is going to like mathematics. (I once received a course critique in which a student wrote, ''All my life I have disliked mathematics and never knew why; now I know why.'' I choose to regard that as a successful course.)

Robert C. Stewart, ''On Teaching Mathematics,'' The Trinity Reporter **10** (2), 16–19 (1979).